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RS-MCB is the simultaneous computer implementation of Ranking and Selection (RS) and Multiple Comparisons with the Best (MCB) procedures. This is made possible by recent developments in statistics which showed that Ranking and Selection (both Subset Selection and Indifference Zone) can be executed simultaneously with Multiple Comparisons with the Best without increasing the error rate of any component inference, for equal as well as unequal sample sizes. These developments are described, and the use of RS-MCB is illustrated with sample computer sessions.

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A COMPUTER PACKAGE FOR RANKING, SELECTION, AND MULTIPLE COMPARISONS WITH THE BEST

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ABSTRACT

RS-MCB is the simultaneous computer implementation of Ranking and Selection (RS) and Multiple Comparisons with the Best (MCB) procedures. This is made possible by recent developments in statistics which showed that Ranking and Selection (both Subset Selection and Indifference Zone) can be executed simultaneously with Multiple Comparisons with the Best without increasing the error rate of any component inference, for equal as well as unequal sample sizes. These developments are described, and the use of RS-MCB is illustrated with sample computer sessions.

1. MULTIPLE DECISIONS AND MULTIPLE COMPARISONS

Multiple Comparisons is concerned with comparing k (≥ 2) treatments (computer systems, sorting algorithms, compilers, etc.). Multiple Decision Theory, in this context, is concerned with making decisions on these k treatments. The two are intimately related: The decision taken by a multiple decision procedure typically corresponds to the inference given by a multiple comparisons procedure. The difference is one of emphasis: Multiple Decisions is more decision or action oriented, whereas Multiple Comparisons is more inference or data analysis oriented.

RS-MCB is the (simultaneous) computer implementation of a particular type of multiple decision procedures, namely, Ranking and Selection procedures, and its associated multiple comparisons method, namely, Multiple Comparisons with the Best.

Multiple Comparisons with the Best (MCB) compares each treatment with the best of the other treatments. Ranking and Selection (RS) decides which treatments can be rejected as the best (Subset Selection), and whether the treatment that appears to be the best can be selected as the best (Indifference Zone selection). Hsu [7] showed that the connection between RS and MCB is as follows. If a treatment is judged worse than the best of the other treatments by MCB, then RS rejects it as the best treatment. On the other hand, if a treatment is judged better than the best of the other treatments by MCB, then RS selects it as the best treatment. Additionally, MCB indicates the magnitude of the difference between each treatment and the best of the other treatments. The theoretical significance of this result is that RS inference (both Indifference Zone and Subset Selection) can be executed simultaneously with MCB inference without increasing the error rate of any of the component inference. The practical implication of this result is that a single computer program suffices for both RS inference and MCB inference.

In Section 2, we describe the statistical inferences that have been implemented in RS-MCB. In Section 3, we give three examples of statistical analysis by the RS-MCB package for different experimental designs.

2. RANKING, SELECTION, AND MULTIPLE COMPARISONS WITH THE BEST

For our general discussion, we consider the balanced oneway design. This is for simplicity of discussion only. As will be illustrated with examples in Section 3, the theory and the computer package are applicable to other designs, balanced or unbalanced. Some more discussion on this is given in Section 2.4.

Let $\pi_1, \pi_2, \ldots, \pi_k$ denote the k treatments and let $\theta_1, \theta_2, \ldots, \theta_k$ denote their respective treatment effects. Suppose under each treatment π_i, a random sample $Y_{i1}, Y_{i2}, \ldots, Y_{in}$ is taken, where between the treatments the random samples are independent. Then under the usual normality and equality of variance assumptions, we have the oneway model

$$Y_{ia} = \theta_i + E_{ia}, \quad i = 1, \ldots, k, \quad a = 1, \ldots, n,$$

where $E_{11}, \ldots, E_{kn}$ are iid normal with mean 0 and variance σ^2 unknown. We use the following notations

$$\bar{Y}_{i.} = \Sigma_{a=1}^{n} Y_{ia}/n, \quad i = 1, \ldots, k,$$

$$s^2 = \mathrm{MSE} = \Sigma_{i=1}^{k} \Sigma_{a=1}^{n} (Y_{ia} - \bar{Y}_{i.})^2/[k(n-1)]$$

for the sample means and the pooled sample variance. For our main discussion, we assume that a LARGER treatment effect implies a better treatment as, for example, in comparing k manufacturing processes in terms of yield. The appropriate modifications when a SMALLER treatment effect implies a better treatment as, for example, in comparing k computer systems in terms of CPU time for similar programs, is indicated in Section 2.3.

Suppose that a larger treatment effect implies a better treatment. For each treatment π_i, consider the quantity $\theta_i - \max_{j\neq i} \theta_j$, which can be termed "Treatment i effect minus the best of the other treatment effects." We claim that, to assess the treatments, very often the parameters $\theta_i - \max_{j\neq i} \theta_j$ for i=1, ..., k are the quantities of primary interest. This can be seen as follows. If $0 \leq \theta_i - \max_{j\neq i} \theta_j$, then treatment π_i is the best, for it is better than the best of the other treatments. If $\theta_i - \max_{j\neq i} \theta_j \leq 0$, then treatment π_i is not the best, for there is another treatment better than it. Even if $\theta_i - \max_{j\neq i} \theta_j \leq 0$, if $-\delta \leq \theta_i - \max_{j\neq i} \theta_j$ where δ is a very small positive number, then treatment π_i is close to being the best. Thus, our statistical inference should concentrate on the parameters $\theta_i - \max_{j\neq i} \theta_j$ for all i.

Given any finite amount of data, due to random fluctuations (noise) in the system, the quantities $\theta_i - \max_{j\neq i} \theta_j$ are not known precisely. Ranking and Selection (RS) takes into account the random fluctuations, and decides which treatments can be REJECTED as the best treatment, and whether the treatment that appears to be the best according to the data can be SELECTED as the best treatment. Multiple Comparisons with the Best (MCB) takes into account the random fluctuations and gives simultaneous UPPER and LOWER BOUNDS on the parameters $\theta_i - \max_{j\neq i} \theta_j$ for all i. We will describe RS and MCB inference in detail below. But from the discussion in the last paragraph, one can already sense that the MCB UPPER bounds will correspond to RS REJECTION inference, and the MCB LOWER BOUNDS will correspond to RS SELECTION inference. This is indeed the case, as weill be seen below. It is easier to describe MCB inference first and then RS inference.

2.1 Multiple Comparisons with the Best (MCB)

Multiple Comparisons with the Best (MCB) obtains, for any user specified confidence level $(1-\alpha)$, a set of $100(1-\alpha)$% simultaneous confidence intervals for $\theta_i - \max_{j\neq i} \theta_j$, of the form

$$[-(\overline{Y}_{i.} - \max_{j\neq i} \overline{Y}_{j.} - D)^-, (\overline{Y}_{i.} - \max_{j\neq i} \overline{Y}_{j.} + D)^+] \tag{2.1}$$

for i = 1, ..., k. Here $-x^- = x$ if x is negative, 0 otherwise; and $x^+ = x$ if x is positive, 0 otherwise. In (2.1), $D = d(\alpha,v)s(2/n)^{1/2}$ where $d(\alpha,v)$ is a critical value that depends on α and v, the degrees of freedom for s^2 (= MSE). The interpretation of these confidence intervals is as follows. For any specified confidence level $(1-\alpha)$ (.95 say), the statement

$$-(\overline{Y}_{i.} - \max_{j\neq i} \overline{Y}_{j.} - D)^- \leq \theta_i - \max_{j\neq i} \theta_j \leq (\overline{Y}_{i.} - \max_{j\neq i} \overline{Y}_{j.} + D)^+$$

holds simultaneously for all i at least $100(1-\alpha)$% (95% say) of the time in the long run upon repetition of the experiment.

The presentation above is based on the latest MCB result, given in HSU [7]. Relevant earlier references are Hsu [4, 5, 6].

2.2 Ranking and Selection (RS)

Ranking and Selection consists of two aspects: Subset Selection, and Indifference Zone Selection.

Subset Selection inference, due to Gupta [2, 3], gives a subset that contains the best treatment. The implied inference is then: Treatments not in Gupta's subset are REJECTED as the best treatment. According to his rule, a treatment π_i is REJECTED as the best treatment if and only if[1]

$$\overline{Y}_{i.} - \max_{j\neq i} \overline{Y}_{j.} + D \leq 0.$$

Comparing with the (2.1), one sees that a treatment is REJECTED if and only if its MCB UPPER BOUND is 0.

Indifference Zone selection, due to Bechhofer [1], appropriately modified for the present setting, SELECTS treatment π_i as the best treatment if and only if

$$0 \leq \overline{Y}_{i.} - \max_{j\neq i} \overline{Y}_{j.} - D.$$

Comparing with (2.1), one sees that a treatment is SELECTED as the best if its MCB LOWER BOUND is 0. Note that, as D is positive for any reasonable α, the treatment with the largest sample mean is SELECTED if that sample mean is significantly larger than the maximum of the other sample means, otherwise no treatment is SELECTED. This last option of no selection is the modification referred to earlier that extends Indifference Zone selection to the present setting of single-stage experiment with variance unknown. See Hsu [4, 5, 6] for more discussions on this.

For each treatment, in addition to reporting whether that treatment is rejected at the chosen confidence level $(1-\alpha)$, it is convenient to report the smallest α for which that treatment can be rejected. This is called the R-value for that treatment. Of course, it would be rather silly to report the R-value of the treatment that appears to be the best. For that treatment, in addition to reporting whether it is selected as the best at the chosen confidence level $(1-\alpha)$, we also report the smallest α for which that treatment can be selected as the best. This is called the S-value of that treatment. Introduced in Hsu [6], it was shown there that R and S-values are particularly suited for computer implementation.

A most important observation to make at this point is that, since the MCB confidence intervals are guaranteed to cover the parameters $\theta_i - \max_{j\neq i} \theta_j$ simultaneously with a probability of at least $(1-\alpha)$, Subset Selection inference and Indifference Zone selection inference can be given simultaneously with a probability of at least $(1-\alpha)$. In fact, since the two aspects of Ranking and Selection correspond to upper and lower MCB bounds, MCB inference and (both aspects of) RS inference can be given simultaneously with the guarantee that ALL the inferences are correct with a probability of at least $(1-\alpha)$. This realization, which came fairly recently (Hsu [4]), made it possible to write a single computer package for Ranking, Selection, and Multiple Comparisons with the Best.

2.3 When Smaller Treatment Effect is Better

Now consider the case where a SMALLER treatment effect implies a better treatment. By symmetry with the earlier discussion, the parameters of primary interest for each treatment π_i is $\theta_i - \min_{j\neq i} \theta_j$, again "Treatment i effect minus the best of the other treatment effects." Now, if $0 \leq \theta_i - \min_{j\neq i} \theta_j$, then treatment π_i is not the best treatment. If $\theta_i - \min_{j\neq i} \theta_j \leq 0$, then treatment π_i is the best treatment. Even if $0 \leq \theta_i - \min_{j\neq i} \theta_j$, suppose $\theta_i - \min_{j\neq i} \theta_j \leq \delta$, where δ is a small positive number, then treatment π_i is close to the best.

MCB inference obtains, for any specified confidence level $(1-\alpha)$, the simultaneous confidence intervals

$$[-(\bar{Y}_{i.} - \min_{j\neq i} \bar{Y}_{j.} - D)^-, (\bar{Y}_{i.} - \min_{j\neq i} \bar{Y}_{j.} + D)^+] \tag{2.2}$$

for $\theta_i - \min_{j\neq i} \theta_j$ for $i = 1, \ldots, k$.

For RS inference, Subset Selection REJECTS treatment π_i as the best treatment if and only if

$$0 \leq \bar{Y}_{i.} - \min_{j\neq i} \bar{Y}_{j.} - D,$$

i.e., when the MCB lower bound for treatment π_i is ≥ 0. Indifference Zone selection inference SELECTS treatment π_i as the best treatment if and only if

$$\bar{Y}_{i.} - \min_{j\neq i} \bar{Y}_{j.} + D \leq 0,$$

i.e., if the MCB upper bound for treatment π_i is ≤ 0. Again, for each treatment except the one that appears to be the best, the R-value is the smallest α for which that treatment can be rejected as best. The S-value for the treatment that appears to be the best represents the smallest α for which it can be selected as best.

2.4 Unbalanced Designs

A great advantage of interfacing the RS-MCB theory with the computer over the usual table look-up method is that the traditionally perceived difficulty with unbalanced (unequal sample sizes) designs disappears. Basically, the traditionally perceived difficulty stemmed from the fact that, for unbalanced designs, the ideal statistical procedure requires a VECTOR of critical values, the dimensions of which equals the number of treatments. As this vector of critical values depends on the sample size configuration, multi-dimensionality precludes any possibility of tabulation. However, for a given data set to be analyzed, the computer can solve for the particular vector of critical values needed AT EXECUTION TIME. This was proposed in Hsu [6] and implemented in the present RS-MCB computer package.

We do not write down all the complicated formulas for unbalanced designs because it is doubtful that they will add much insight. The RS-MCB computer package in fact implements the general formulas for unbalanced designs, which reduce to the formulas given above in the balanced case. Thus, in terms of computer implementation, balanced or unbalanced design really makes no difference.

3. EXAMPLES OF USING THE RS-MCB PACKAGE

Output from RS-MCB consists of two parts: Ranking and Selection (Rejection and Selection), and Multiple Comparisons with the Best. In the Ranking and Selection portion, an "*" in the REJECT column means that treatment is rejected as the best by Subset Selection (i.e., excluded from Gupta's subset). The rejected treatments are exactly those with R-values less than α. An "*" in the SELECT column means that treatment is select as the best by the (modified) Indifference Zone selection rule. The treatment that appears to be the best is selected if and only if its S-value is less than α. In the Multiple Comparisons with the Best portion, in addition to the numerical values of MCB upper and lower bounds for treatment minus best of other treatments, these confidence intervals are plotted. If a larger treatment effect implies a better treatment, then a confidence interval more to the RIGHT implies a better treatment. Conversely, if a smaller treatment effect implies a better treatment, then a confidence interval more to the LEFT implies a better treatment.

3.1 Balanced Oneway Design

Suppose five treatments are being compared, and that a larger treatment effect implies a better treatment. Independent random samples have been taken from the five treatments, with the following result.

i	Y_{i1}	Y_{i2}	Y_{i3}	$\bar{Y}_{i.}$
1	42	35	37	38
2	36	34	32	34
3	36	27	30	31
4	35	27	28	30
5	22	19	13	18

For this data set, s^2 = MSE = 15.6, with associated degrees of freedom $\nu = 5(3-1) = 10$. Suppose we choose confidence level $(1-\alpha) = .95$, i.e., $\alpha = .05$. An interactive RS-MCB session would then proceed as in Figure 1 below.

First consider inference on treatments that appear to be inferior. For treatments 4 and 5, the associated R-values are less than $\alpha = .05$. Thus, as indicated by the "*" in the REJECT column, these treatments can be REJECTED as the best treatment at $\alpha = .05$. That is, they are excluded from Gupta's subset at $\alpha = .05$. Note that the MCB upper bound on $(\theta_i - \max_{j\neq i} \theta_j)$ are zero for these two treatments, indicating that for each of these two treatments there is another treatment better than it, agreeing with the conclusions reached by the R-values. Treatments 2 and 3 have R-values greater than $\alpha = .05$. Thus, as indicated by the absence of "*" in the REJECT column, we are unable to reject these treatments as the best at $\alpha = .05$. That is, together with Treatment 1, they would be included in Gupta's subset at $\alpha = .05$. Note that their associated MCB confidence interval for $(\theta_i - \max_{j\neq i} \theta_j)$ cover 0, indicating that indeed one of them may be the best treatment.

Next consider infernece on Treatment 1, the treatment that appears to be the best. Since its S-value is greater than $\alpha = .05$, we are unable to SELECT Treatment 1 as the best at $\alpha = .05$. Note, however, its MCB lower bound on $(\theta_i - \max_{j\neq i} \theta_j)$ is relatively close to 0. If a treatment effect within 4 units of the best could be considered "good enough," for example, then we would be able to declare Treatment 1 "good enough" at $\alpha = .05$.

We emphasize again that all the inferences are guaranteed to be correct simultaneously with a probability of at least 0.95.

```
------------------
RS-MCB VERSION 7X8
------------------

INPUT RUN NAME
winter simulation 84 balanced oneway example
BEST TREATMENT LARGEST OR SMALLEST?
largest
INPUT CONFIDENCE LEVEL DESIRED
0.95
INPUT NUMBER OF TREATMENTS
5
INPUT TREATMENT ID, SAMPLE SIZE, SAMPLE MEAN
1 3 38.
2 3 34.
3 3 31.
4 3 30.
5 3 18.
INPUT DEGREES OF FREEDOM, MSE
10 15.6

CONFIDENCE COEFFICIENT = 0.9500
```

REJECTION AND SELECTION

TREATMENT	SAMPLE MEAN	SAMPLE SIZE	R-VALUE	REJECT	S-VALUE	SELECT
1	38.0000	3			0.2956	
2	34.0000	3	0.2956			
3	31.0000	3	0.0798			
4	30.0000	3	0.0488	*		
5	18.0000	3	0.0002	*		

MULTIPLE COMPARISONS WITH THE BEST

TREATMENT	SAMPLE MEAN	SAMPLE SIZE	TREATMENT MINUS BEST OF OTHER TREATMENTS LOWER BOUND	UPPER BOUND
1	38.0000	3	-3.9526	11.9526
2	34.0000	3	-11.9526	3.9526
3	31.0000	3	-14.9526	0.9526
4	30.0000	3	-15.9526	0.0000
5	18.0000	3	-27.9526	0.0000

```
                         TREATMENT MINUS BEST OF OTHER TREATMENTS
                                        (*1.0E  0)
TREATMENT      -30.0     -20.0     -10.0      0.0       10.0

               ---+---------+---------+---------!---------+-------
    1                                       (---!+++*+++++++)
    2                                   (-------*---!+++)
    3                                (-------*------!)
    4                               (-------*-------)
    5                   (-------*-------------------)
               ---+---------+---------+---------!---------+-------
               -30.0     -20.0     -10.0      0.0       10.0
```

Figure 1. Computer Session for Balanced Oneway Design

3.2 Unbalanced Oneway Design

Suppose four treatments are being compared, and a smaller treatment effect implies a better treatment. Independent random samples have been taken from the four treatments, with the following result:

i	Y_{i1}	Y_{i2}	Y_{i3}	$\bar{Y}_{i.}$
1	36	32	-	34
2	36	-	30	33
3	-	36	28	32
4	22	19	13	18

In the above table, a "-" indicates a missing value.

For this data, s^2 = MSE = 20 with associated degrees of freedom $v = 1+1+1+2 = 5$. Suppose we choose confidence level $(1-\alpha) = 0.975$, then an interactive RS-MCB session would proceed as in Figure 2 below.

The R-values for treatments 1, 2, and 3 are all less than $\alpha = 0.025$. Thus, as indicated by the corresponding "*" in the REJECT column, all three are rejected as the best at $\alpha = 0.025$, i.e., only Treatment 4 remains in Gupta's subset. Treatment 4 has an S-value less than $\alpha = 0.025$. Thus, as indicated by the "*" in the SELECT column, Treatment 4 is selected as the best by Indifference Zone selection, which of course agrees with the result given by Subset Selection. Notice that the MCB lower bounds on $\theta_i - \min_{j\neq i} \theta_j$ are 0 for treatments 1, 2, and 3, indicating that for each of these treatments there is another better than it, agreeing with the conclusion reached by the R-values. The MCB upper bound on $\theta_i - \min_{j\neq i} \theta_j$ is 0 for Treatment 4, indicating that it is better than the best of the other treatments, agreeing with the conclusion reached by the S-value.

We again emphasize that all the inferences are guaranteed to be correct simultaneously with a probability of at least 0.975.

```
------------------
RS-MCB VERSION 7X8
------------------

INPUT RUN NAME
winter simulation 84 unbalanced oneway example
BEST TREATMENT LARGEST OR SMALLEST?
smallest
INPUT CONFIDENCE LEVEL DESIRED
0.975
INPUT NUMBER OF TREATMENTS
4
INPUT TREATMENT ID, SAMPLE SIZE, SAMPLE MEAN
1 2 34.
2 2 33.
3 2 32.
4 3 18.
INPUT DEGREES OF FREEDOM, MSE
5 20.

CONFIDENCE COEFFICIENT = 0.9750
```

REJECTION AND SELECTION

TREATMENT	SAMPLE MEAN	SAMPLE SIZE	R-VALUE	REJECT	S-VALUE	SELECT
1	34.0000	2	0.0130	*		
2	33.0000	2	0.0166	*		
3	32.0000	2	0.0215	*		
4	18.0000	3			0.0215	*

MULTIPLE COMPARISONS WITH THE BEST

TREATMENT	SAMPLE MEAN	SAMPLE SIZE	TREATMENT MINUS BEST OF OTHER TREATMENTS LOWER BOUND	UPPER BOUND
1	34.0000	2	0.0000	29.6730
2	33.0000	2	0.0000	28.6730
3	32.0000	2	0.0000	27.6730
4	18.0000	3	-27.6730	0.0000

```
                TREATMENT MINUS BEST OF OTHER TREATMENTS
                               (*1.0E  1)
TREATMENT      -4.0      -2.0       0.0       2.0       4.0
           ----+---------+---------!---------+---------+------
    1                              (+++++++*++++++)
    2                              (+++++++*+++++)
    3                              (++++++*++++++)
    4                    (------*------)
           ----+---------+---------!---------+---------+------
               -4.0      -2.0       0.0       2.0       4.0
```

Figure 2. Computer Session for Unbalanced Oneway Design

3.3 Randomized Complete Blocks

Consider data in the form of a randomized complete block design with four treatments and three blocks, as follows:

j/i	1	2	3	4	$\overline{Y}_{.j}$
1	1.	2.	3.	6.	3.
2	1.	5.	6.	8.	5.
3	4.	5.	9.	10.	7.
$\overline{Y}_{i.}$	2.	4.	6.	8.	

In the table above, i indexes treatments and j indexes blocks. We assume the usual additive (no interaction) model, and that comparisons among the treatments are of interest, not the blocks. Skipping over the theoretical details that can be found in [5], to compare the treatments, one would proceed as before, using $\overline{Y}_{i.}$, except now s^2 = MSE must be computed appropriately for this model. For this data, one finds s^2 = MSE = 1.0 with associated degrees of freedom (4-1)*(3-1) = 6. Suppose a larger treatment effect implies a better treatment, then for $(1-\alpha) = 0.95$, an interactive RS-MCB session would proceed as in Figure 3 below.

Treatments 1 and 2 have R-values less than $\alpha = 0.05$. Thus, they are rejected as the best, and only Treatments 3 and 4 remain in Gupta's subset for the best treatment. As the S- (R-) value of Treatment 4 (3) is greater than $\alpha = 0.05$, we are unable to select (reject) Treatment 4 (3) as the best at $\alpha = 0.05$. However, as indicated by the closeness to 0 of its MCB lower bound, we can assert that Treatment 4 is close to the best.

```
------------------
RS-MCB VERSION 7X8
------------------

INPUT RUN NAME
winter simulation 84 randomized complete blocks example
BEST TREATMENT LARGEST OR SMALLEST?
largest
INPUT CONFIDENCE LEVEL DESIRED
0.95
INPUT NUMBER OF TREATMENTS
4
INPUT TREATMENT ID, SAMPLE SIZE, SAMPLE MEAN
1 3 2.
2 3 4.
3 3 6.
4 3 8.
INPUT DEGREES OF FREEDOM, MSE
6 1.

CONFIDENCE COEFFICIENT = 0.9500

                             -----------------------
                             REJECTION AND SELECTION
                             -----------------------
TREATMENT   SAMPLE MEAN   SAMPLE SIZE   R-VALUE   REJECT   S-VALUE   SELECT
---------   -----------   -----------   -------   ------   -------   ------
    1          2.0000          3        0.0004      *
    2          4.0000          3        0.0034      *
    3          6.0000          3        0.0575
    4          8.0000          3                             0.0575
```

```
                  ---------------------------------
                  MULTIPLE COMPARISONS WITH THE BEST
                  ---------------------------------

                            SAMPLE    TREATMENT MINUS BEST  OF OTHER TREATMENTS
TREATMENT   SAMPLE MEAN      SIZE             LOWER BOUND  UPPER BOUND
---------   -----------     ------            -----------  -----------
    1          2.0000          3                 -8.0892       0.0000
    2          4.0000          3                 -6.0892       0.0000
    3          6.0000          3                 -4.0892       0.0892
    4          8.0000          3                 -0.0892       4.0892

                  TREATMENT MINUS BEST OF OTHER TREATMENTS
                                (*1.0E  0)
TREATMENT         -10.0     -5.0       0.0       5.0       10.0
            ---------+---------+---------!---------+---------+-
    1                      (---*-----------)
    2                          (---*-------)
    3                              (---*---)
    4                                      (+++*+++)
            ---------+---------+---------!---------+---------+-
                 -10.0     -5.0       0.0       5.0       10.0
```

Figure 3. Computer Session for Randomized Complete Blocks

4. COMPUTER IMPLEMENTATION CONSIDERATIONS

The RS-MCB computer package, written in ANSI FORTRAN, is being distributed on a nonprofit basis. It is available on magnetic tape or can be sent through BITNET upon request. Either a FORTRAN 66 version or a FORTRAN 77 version can be specified. It has been tested on AMDAHL 470, DEC20, and IBM 4341, using the IBM VS compiler and the DEC FORTRAN 77 compiler. A 50-page User's Guide accompanies the package.

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